a review of Cartan for beginners: Differential geometry via moving frames and exterior differential systems by Ivey, Thomas A.; Landsberg, J. M.

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Cartan for beginners: Differential geometry via moving frames and exterior differential systems. (English) Zbl 1105.53001

Graduate Studies in Mathematics 61. Providence, RI: American Mathematical Society (AMS) (ISBN 0-8218-3375-8/hbk). xiv, 378 p. (2003).

This book is devoted to the study of geometry and partial differential equations by making use of moving frames and exterior differential systems. It consists of 8 chapters, the core of which is Chapter 4, 5 and 7.

Chapter 1 motivates the use of differential forms to study problems in geometry and partial differential equations. Chapter 2 covers traditional material, namely, the geometry of surfaces in three-dimensional Euclidean space, submanifolds of higher-dimensional Euclidean space, and the rudiments of Riemannian geometry. The presentation emphasizes finding and interpreting differential invariants, concentrating mainly on local geometry. Chapter 3 is an updated version of [*P. Griffiths* and *J. Harris*, Ann. Sci. Éc. Norm. Supér. (4) 12, 355–452 (1979; Zbl 0426.14019)], studying the local geometry of submanifolds of projective space and applications to algebraic geometry. More material is published by the second author's [Algebraic geometry and projective differential geometry. Seoul: Seoul National University (1999; Zbl 1012.14015)]. Chapters 4, 5 and 7 deal with the Cartan algorithm and Cartan-Kähler theorem. The general version of Cartan's test is presented in Chapter 7, while Chapter 5 is concerned with the Cartan algorithm for linear Pfaffian systems. Chapter 4 investigates constant coefficient homogeneous systems of partial differential equations and the linear algebra associated to the corresponding exterior differential systems. Chapter 6 is a detour to the classical theory of characteristics, Darboux's method for solving partial differential equations, and Monge-Ampère equations in modern language. Chapter 8 is a leisurely presentation of G-structures and connections.

The book is a well-written textbook for a graduate-level course. The book is intended for two kinds of audience, namely, graduate students who have some familiarity with classical differential geometry and differential manifolds and experts in such areas as the theory of partial differential equations and algebraic geometry who would like to learn how moving frames and the techniques of exterior differential systems apply to their own fields. The book prepares the reader for such an advanced textbook on exterior differential systems as [*R. L. Bryant, S. S. Chern, R. B. Gardner, H. L. Goldschmidt* and *P. A. Griffiths*, Exterior differential systems. New York etc.: Springer-Verlag (1991; Zbl 0726.58002)].

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MSC:

53-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to differential geometry
58A15	Exterior differential systems (Cartan theory)
53A20	Projective differential geometry
53C10	G-structures
53C05	Connections, general theory
34C41	Equivalence and asymptotic equivalence of ordinary differential equations

Keywords:

moving frames; G-structures; projective differential geometry; Cartan's method of equivalence; algebraic geometry; exterior differential systems; Cartan algorithm; Cartan-Kähler theorem; Cartan's test; connection; Cauchy-Kowalevski theorem; Frobenius theorem; scalar curvature; Riemannian curvature tensor; constant mean curvature; symplectic form; torsion; Veronese re-embedding; Monge-Ampère equations; Pfaffian system; Darboux's method; Zak and Landman theorems; jet; sine-Gordon representation; Weierstrass representation